A Problem-Based Learning Approach to Mathematics Support?

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Abstract

This paper describes a ‘resource-based’ mathematics service programme for Physics students at the University of Leicester which has run since 1985. In addition it outlines a pilot project to use Problem-Based Learning (PBL) to address some of the issues raised by this programme. It concludes that whilst this limited experiment indicates that PBL does not help significantly in improving mathematical skills, the project has suggested some future developments.

Background

The Department of Physics at Leicester took over responsibility for the teaching of its ‘service course’ in mathematical techniques in 1984. One of the authors of this paper (DJR) led a teaching team of six academic staff from Mathematics and from Physics with backgrounds in mathematics with the ringing endorsement from colleagues that “we couldn’t make matters any worse”. The approach adopted several new features which will be discussed more fully below:

- No lectures: informal observations of student responses to questions during lectures led us to believe that the attention span of our typical undergraduate in a mathematics lecture was between five and ten minutes. Apart from its social aspect, the typical lecture was therefore a sparsely attended, error prone, dictation session. Rather than change the lecture, it was decided that the time of the teaching team would be better spent interacting with students in workshops and tutorials. More recently the ‘no lectures’ rule has been relaxed to include one lecture a week to introduce the material and provide help with reading the more difficult sections;
- A specially written text;
- Flexible pacing as a response to the dispersion of prior learning on entry.

Generative Mathematics

The standard format for the presentation of mathematics is theorem, proof, example, exercise. The example is usually a particular application of the general result stated in the theorem and then demonstrated in the proof, and the exercise is designed to allow the students to practice applying the theorem for themselves. Unfortunately the proof very often fails to illuminate why the theorem is really true: each step is seen to follow the previous step without providing any broader sense of where it is going. In any event, students do not learn the proofs but attempt to memorise the results in case they come up in the examination.

The sequence theorem-proof-example is not how mathematical results are typically generated (Burn, 2002): a general result emerges from a number of specific examples. Moreover, well-chosen examples often illustrate why the result is true. Thus, for the
Table 1. Marks distribution on the two main techniques examination papers for the 2007 cohort showing the division into three groups. The majority of the failing students pass the re-sit examination.

purpose of a course in mathematical techniques for Physics, we can start with an example and follow this with an exercise. A well-chosen exercise will not be solvable by ‘plug and chug’ from the example, but will require an insight into how the example works. In some cases we might want to add the theorem as a reasonable generalisation; indeed in many cases the proof might be a generalisation of an example.

The Text

Since the text is not a “self-study” book it does not need to examine every potential confusion or every possible variation on a theme or go into a lot of background. The text covers all of the standard mathematics for Physics in 15 brief chapters each of approximately 15 pages. This provides a manageable quantity of reading and initiates the process of ‘chunking’ of mathematical knowledge.

Observations in tutorials have shown that many students enter university not knowing how to read mathematics. The text adopts a two-column format in which the formal mathematics appears in the right hand column and the thought processes behind the mathematics appear alongside in the left hand column.

Flexible Pacing

It is our belief that streaming students on the basis of their prior learning has a negative effect on motivation. Rather, each topic can be studied in either one or two weeks allowing both a difference in contact time and a quantifiable progression rate through the programme. The gaps created by progressing more rapidly are filled by higher-level work; by taking this more advanced core material early students create space for additional options and hence the opportunity to obtain a better class of degree.

Outcomes

Students attend one workshop and one tutorial per week and must submit for each a piece of work to be marked by the tutor as well as a multiple-choice paper to be marked by computer. In addition there are two end of term ‘open book’ test papers and ten hours of examinations. This level of assessment means that students must attempt all parts of the course. In addition it means that risk of failure is highly visible.
Interestingly, as illustrated in Table 1, the overall marks are tri-modal: one group of students is not significantly stretched by this material; a second group fail multiple modules; and a third group lies in between. Of the 1200 or so students who have been through the course, only 3 have ultimately not progressed because of a failure in the mathematics component alone.

Evaluation

Focus groups and observation of students have been used to obtain feedback on the course. The main issues that have emerged from the point of view of this paper are firstly that the programme is somewhat boring and secondly that many students still have only a limited ability applying mathematics in novel contexts in later work.

Problem-Based Learning

A major component of the Physics programme is taught through Problem-Based Learning (PBL) (Raine & Symons, 2005). In addition the Interdisciplinary Science programme is taught entirely by PBL with the exception of the skills component (which includes IT and mathematics). PBL is intended both to improve motivation by arousing student interest and to embed knowledge more securely by adopting a research approach. The question therefore arises as to whether PBL can be used in the context of service mathematics (Raine & Symons, 2006). To investigate this a PBL version of the first year mathematics service course for Year 1 Interdisciplinary Science students was prepared, hoping to address the following research questions:

- Is low performance in Mathematics amongst entrants to science degrees influenced by a negative attitude towards the subject?
- Can we change attitudes towards mathematics through PBL?
- Does an improved attitude towards mathematics coincide with an improvement in performance?

The number of students on the Interdisciplinary Science programme has been small (entries of 6, 12, 4, and 16 over the four years to date). It is unfortunate that the year in which the PBL programme was introduced coincided with the smallest entry of just 4 students. Nevertheless, the four students spanned a range of prior learning and attitudes and the research techniques used appear to have provided some useful results.

Pilot Evaluation Strategy

The pilot evaluation strategy had four components. First, to establish a baseline of mathematics knowledge, the students took a timed unseen ‘test’ prior to any teaching. Second, to establish a baseline for attitudes towards mathematics, students were interviewed as a group and their responses to a set of questions recorded. Finally, at the end of the year students were given a post-test with questions covering the same material as the pretest and, additionally, changes in attitudes were investigated through a second group interview.
Materials for PBL

Two types of material were developed as part of the pilot project: PBL questions and online videos. The PBL questions were sets of problems that formed the basis of weekly class meetings lasting 1 to 1.5 hours. The facilitator guided students through the knowledge they would need to tackle the problems and answered questions on this material. Standard textbooks were used and students were expected to complete the problems outside of the class.

Additionally, a set of short videos covering individual topics was produced: since each video addressed a single issue they were each no more than 5 minutes in length. The videos were made in a tutorial setting with the tutor interacting with a pair of students. This was intended to ensure that the viewer felt part of the group and was not being spoken at by a ‘talking head’. It also enabled natural pauses during which the viewer could think alongside the video tutees. To allow single ‘takes’ we used a two camera set-up even though this added to the editing overheads. In addition we used an interactive whiteboard to record the mathematical writing directly and clearly; this also required editing. Our philosophy was to encourage students to view mathematics (at this level) as an encoding of what they already know. Hence, the commentary is correspondingly informal and more appropriate to a ‘live’ video presentation than a text.

The videos were placed on the University Virtual Learning Environment (VLE); unfortunately it was not possible to monitor their use since the statistics of site visits from the VLE are misleading in that they overestimate the number of actual viewings of the material. The videos are publicly available on the πCETL web site.

Pilot Evaluation

The points to emerge are as follows (Barker, 2008):

• There was some increase in positive attitudes toward mathematics amongst all four students who also expressed a greater confidence in tackling problems;
• There was some improvement in mathematics performance by the weaker of the students (+34% overall but the increase from 29% to 39% in average student marks is disappointing);
• There was a general dislike of the PBL approach to provide supporting skills. Students all felt that their main modules involved a constant diet of PBL; they wanted the mathematics classes to help them directly to tackle the problems they already had, not to add more problems;
• The video support materials were found to be useful.

The small improvements in attitude and ability are more likely therefore to have resulted from the attention they received as a small group than from the PBL approach.

Course Development

While the specific questions asked by the Project have received somewhat negative answers, the results of the Project have been greatly beneficial in several ways. Firstly, the materials from the Project will be embedded in the course: the videos are independent of the PBL approach and will be used to support both Physics and Interdisciplinary Science;
the PBL problems will be adapted as exercises to reinforce learning, especially for the stronger students. Secondly, the mathematics support course for Interdisciplinary Science has been redesigned in a novel way. A number of exercises on a given topic are set at the start of each week. Students who obtain full marks on the exercises are excused from mathematics classes. Other students attend an initial class where the material is explained. They then submit their attempts at the problems. There is then a second class where they are given feedback on their attempts and further assistance. The submission process and written feedback continues (in theory) until students can demonstrate competence (arbitrarily chosen as greater than 80%). This is reflected in the marking scheme which is 0 for each weekly unit until competence is achieved. This approach will be formally evaluated at the end of the session but attendance and submission rates as well as anecdotal evidence suggest that it is working very well.

Our conclusion for the mathematical techniques course for Physics students is that we should eschew any wholesale redesign along PBL lines, but that we should endeavour to link future mathematics teaching to existing PBL components and evaluate the results.

References


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